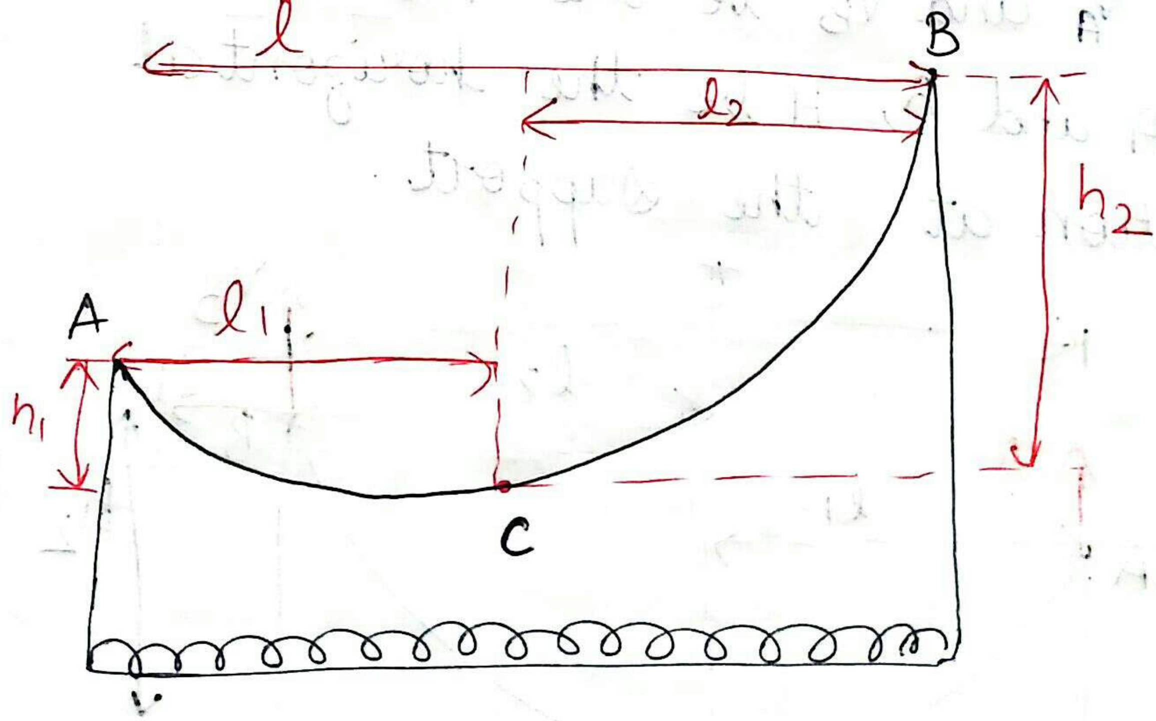
CABLE WITH ENDS AT DIFFERENT LEVELS

Consider a cable which is supported at A and B. A is h_1 metres above the lowest point C and B is h_2 metres above C. The cable is subjected to a uniformly distributed load w per unit horizontal length over entire span l.

Let the horizontal distance between A and C be l_1 and that between C and B be l_2. Let the horizontal reaction at supports be H.

here $l_1 + l_2 = l$



Since ACB is parabolic, we have

$$\frac{x^2}{y} = \text{constant} \quad \text{where } C \text{ is origin}$$

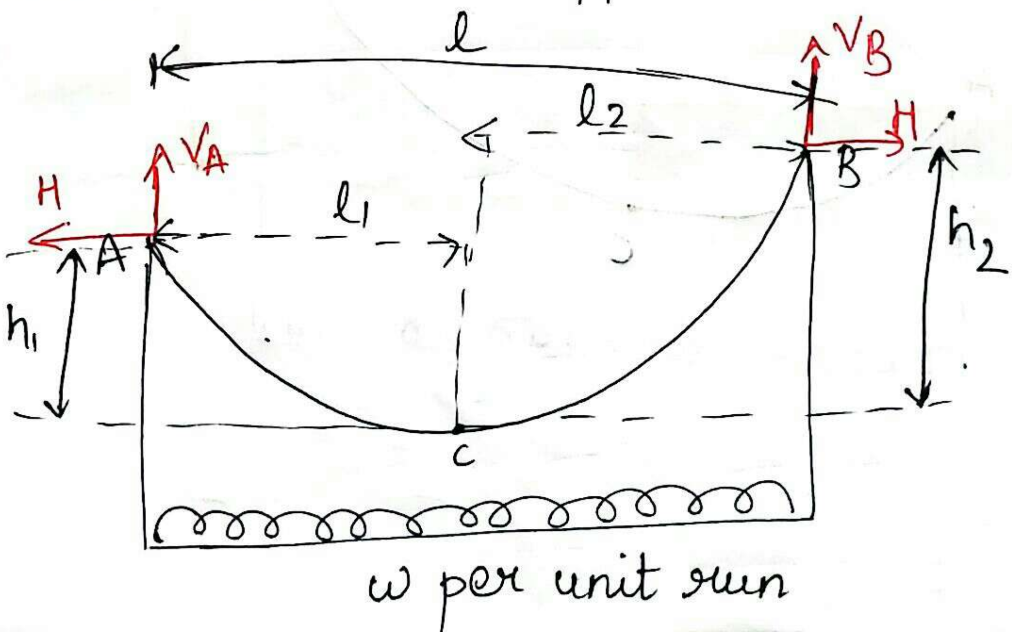
$$\therefore \frac{l_1^2}{h_1} = \frac{l_2^2}{h_2}$$

$$\therefore \frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}}$$

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}}$$

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \frac{l}{\sqrt{h_1} + \sqrt{h_2}}$$

Let V_A and V_B be the vertical reactions at A and B, H be the horizontal reaction at the support.



$\Sigma F_y = 0$

$V_A + V_B = wl$

Taking moments about C of the forces on the left hand side C, we have.

~~$V_A \times l_1 + H \times$~~ $(V_A \times l_1) - (H \times h_1) - w \times l_1 \times \frac{l_1}{2} = 0$

$(V_A \times l_1) - (H \times h_1) - w\frac{l_1^2}{2} = 0$

$V_A = \frac{H \times h_1}{l_1} + \frac{wl_1}{2} \rightarrow (1)$

Similarly taking moments about C of the forces on the right hand side, we have

$-(V_B \times l_2) + (H \times h_2) + w \times l_2 \times \frac{l_2}{2} = 0$

$(V_B \times l_2) = (H \times h_2) + w \times \frac{l_2^2}{2}$

$V_B = \frac{Hh_2}{l_2} + \frac{wl_2}{2} \rightarrow (2)$

Adding equation (1) and (2)

$V_A + V_B = \frac{Hh_1}{l_1} + \frac{wl_1}{2} + \frac{Hh_2}{l_2} + \frac{wl_2}{2}$

$= H\left[\frac{h_1}{l_1} + \frac{h_2}{l_2}\right] + \frac{w}{2}(l_1 + l_2)$

Since $V_A + V_B = wl$ from $\Sigma F_y = 0$

$$wl = H\left[\frac{h_1}{l_1}+\frac{h_2}{l_2}\right] + \frac{wl}{2}$$

$$H = \frac{\frac{wl}{2}}{\left[\frac{h_1}{l_1}+\frac{h_2}{l_2}\right]}$$

Putting $l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$ & $l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$ we hav

$$\therefore H = \frac{\frac{wl}{2}}{\frac{h_1(\sqrt{h_1}+\sqrt{h_2})}{l\sqrt{h_1}} + \frac{h_2(\sqrt{h_1}+\sqrt{h_2})}{l\sqrt{h_2}}}$$

$$\therefore H = \frac{\frac{wl}{2}}{\frac{h_1\times\sqrt{h_1}(\sqrt{h_1}+\sqrt{h_2})}{l\sqrt{h_1}\times\sqrt{h_1}} + \frac{h_2\times\sqrt{h_2}(\sqrt{h_1}+\sqrt{h_2})}{l\times\sqrt{h_2}\times\sqrt{h_2}}}$$

$$H = \frac{\frac{wl}{2}}{\frac{\sqrt{h_1}(\sqrt{h_1}+\sqrt{h_2})}{l} + \frac{\sqrt{h_2}(\sqrt{h_1}+\sqrt{h_2})}{l}}$$

$$H = \frac{\frac{wl^2}{2}}{(\sqrt{h_1}+\sqrt{h_2})(\sqrt{h_1}+\sqrt{h_2})}$$

$$\therefore H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$

Length of cable

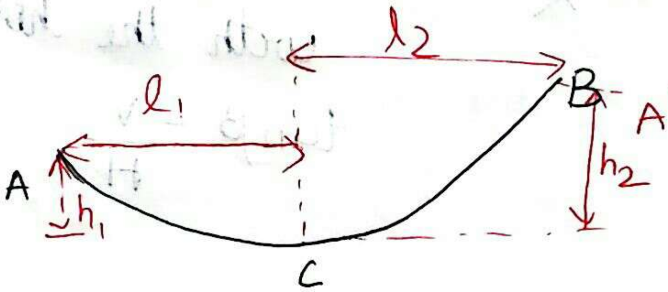
$$L = \frac{1}{2}\left[\text{sum of } (ACB' \;\&\; A'CB)\right]$$

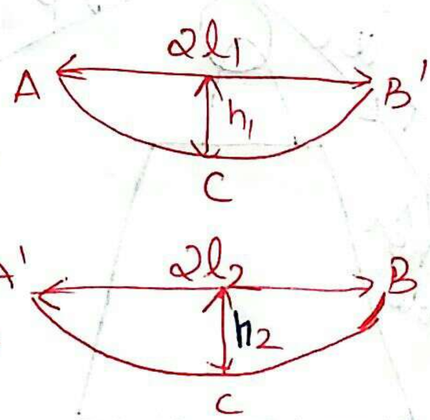
$$= \frac{1}{2}\left[2l_1 + \frac{8}{3}\times\frac{h_1^2}{2l_1} + 2l_2 + \frac{8}{3}\times\frac{h_2^2}{2l_2}\right]$$

$$= l_1 + \frac{2h_1^2}{3l_1} + l_2 + \frac{2h_2^2}{3l_2}$$

$$= (l_1+l_2) + \frac{2h_1^2}{3l_1} + \frac{2h_2^2}{3l_2}$$

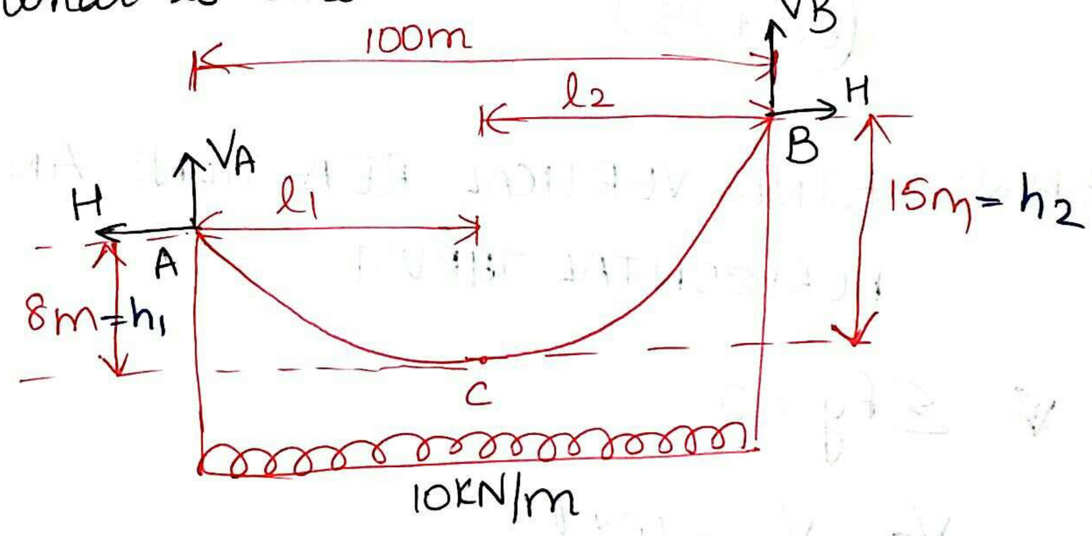
$$L = l + \frac{2h_1^2}{3l_1} + \frac{2h_2^2}{3l_2}$$





PROBLEMS

1) A cable of span 100m has its ends at height 8m and 15m, above the lowest point of cable. It carries a uniformly distributed load of 10KN/m. Determine the horizontal and vertical reactions. What is the length of the cable? Where & what is the max tension in cable?



Given $h_1 = 8m$

$h_2 = 15m$

$w = 10 KN/m$

STEP 1:- TO FIND THE LENGTHS l_1 & l_2

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}}$$

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \frac{l}{\sqrt{h_1}+\sqrt{h_2}}$$

$$\frac{l_1}{\sqrt{8}} = \frac{l_2}{\sqrt{15}} = \frac{100}{\sqrt{8}+\sqrt{15}}$$

$$l_1 = \frac{\sqrt{8} \times 100}{(\sqrt{8}+\sqrt{15})} = 42.206\,m$$

$$l_2 = \frac{\sqrt{15} \times 100}{(\sqrt{8}+\sqrt{15})} = 57.79\,m.$$

STEP 2:- FIND VERTICAL REACTIONS AND HORIZONTAL THRUST

$\Sigma F_y = 0$

$V_A + V_B = w \times l$

$V_A + V_B = 10 \times 100$

Taking moment about A

~~consider forces on the left side~~

~~$V_a \times l_1$~~ $\Sigma M_A = 0$

$\Rightarrow -V_b \times l + H(15-8) + \left(10 \times \frac{100 \times 100}{2}\right) = 0$

$\Rightarrow -V_b \times l + H(7) + 50{,}000 = 0 \rightarrow (1)$

Now take moments about C

$\Sigma M_C = 0$

Consider forces on the right side

$(-V_b \times l_2) + (H \times 15) + 10 \times \frac{57.79 \times 57.79}{2} = 0$

$(-V_b \times 57.7) + 15H + 16698.42 = 0 \rightarrow (2)$

Substituting (2) in (1)

Equation (2) $\Rightarrow \frac{(V_b \times 57.7) - 16698.4}{15} = H$

$\Rightarrow$ Now substitute the above value of H in (1)

$(-V_b \times 100) + 7H + 50{,}000 = 0$

$(-V_b \times 100) + 7\left[\frac{V_b \times 57.7 - 16698.4}{15}\right] + 50{,}000 = 0$

$-100 V_b + 26.92 V_b - 7792.5 + 50{,}000 = 0$

$73 V_b = 42207.5$

$V_b = 578.18 \text{ KN}$

Sub V_b in (2) $\Rightarrow H = \frac{(578.18 \times 57.7) - 16698.4}{15}$

$= 1110.83 \text{ KN}$

Now, $V_A + V_B = 1000$

$$V_A = 1000 - V_B$$

$$= 1000 - 578.18$$

$$= 421.82 \text{ KN}.$$

STEP 3:-

Length of the cable.

$$L = l + \frac{2}{3}\frac{h_1^2}{l_1} + \frac{2}{3}\frac{h_2^2}{l_2}$$

$$= 100 + \left(\frac{2}{3}\frac{\times 8^2}{42.2}\right) + \frac{2}{3}\frac{\times 15^2}{57.7}$$

$$= 103.6 \text{ m}$$

STEP 4:- Maximum Tension in cable

Since $V_B > V_A$, the max tension occurs at V_B

$$\therefore T_{max} = \sqrt{V_B^2 + H^2}$$

$$= \sqrt{578.18^2 + 1110.83^2}$$

$$= 1252.29 \text{ KN}.$$

② A cable is suspended from the points A and B which are 80m apart horizontally and are at different levels, the point A being 5m vertically higher than the point B and the lowest point in the cable is 10m below A.. The cable is subjected to a UDL of 30 KN/m over the horizontal span. Determine the horizontal and vertical reactions at each end and also max tension in the cable?